Guidelines and Specifications for Global Navigation Satellite System Land Surveys in Connecticut

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prepared by

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Purpose

The purpose of this document is to recommend specifications for performing land surveys using Global Navigation Satellite System (GNSS) positioning, in whole or in part, that comply with minimum accuracy standards, classifications, and content requirements of Land Surveys set forth in the Regulations of Connecticut State Agencies, Sections 20-300b-1 through 20-300b-20.

It is important to establish guidelines for conducting land surveys using measurements derived from GNSS observations because positioning with GNSS technologies has become mainstream in the land surveying profession,

Scope

This document is only concerned with relative positioning, using phase differenced observables resulting in centimeter-level accuracies when properly referenced to the National Spatial Reference System (NSRS). Only phase differenced observations whose resulting coordinates are properly referenced, including those being referenced to a local control survey, are acceptable for surveying positioning.

Observations and Equipment

- Carrier phase shall be observed. Doppler and timing codes (e.g., GPS C/A and P-codes) may be observed.
- L1-only receivers are acceptable only for baselines less than 10 km (6.2 mi) in length.
- Fixed height tripods are recommended for vertical control surveys.
- Geodetic-grade antennas shall be used.

Procedures

There shall be favorable ionospheric, tropospheric, and multipath conditions.

Ionospheric conditions can be assessed by consulting the National Space Weather Service (http://www.swpc.noaa.gov/) and poor conditions avoided by not observing during severe space weather.

Poor tropospheric conditions can be avoided by not positioning while weather fronts pass through.

Sites that are partially obscured by nearby objects such as vegetation, buildings, and hills are prone to producing multipath. Such sites are to be avoided if possible.

Mission planning can inform the surveyor when the best conditions (e.g., low DOP, maximum satellite availability) will occur and aid in effective time management. Mission planning is especially important for suboptimal sites, to ensure quality measurements can be made efficiently.

For each observed point in a static or rapid-static survey, field notes should minimally include a description of the marker with sketch and/or photographs, date of occupation, start and stop times, antenna type and height (noted as slant or vertical), equipment serial numbers, and weather conditions.

Redundancy

Accuracy in GNSS surveys is assessed through redundancy. Redundancy occurs in several ways including occupation duration, use of multi-frequency/multi-constellation receivers, and reoccupation.

- Occupation duration: For static or rapid-static occupations, the longer the time between the epochs, the greater the information in those data. For example, 60 epochs collected over 600 seconds will result in higher accuracy than 60 epochs collected over 60 seconds.
- Multi-constellation receivers: Observables from five or more NAVSTAR GPS SV's, observed simultaneously, shall be used to process a baseline. Additional data from other constellations, such as GLONASS, are beneficial, but not required. *Data from other constellations shall not be used as a substitute for the required NAVSTAR GPS SV's*.
- Multi-frequency receivers: Observables from two or more frequencies (L1, L2, L5, etc.) provide redundant information generally resulting in better differencing results and ionospheric delay removal.
- Reoccupation: The geometry of the SV's in view directly affects position quality. Reoccupation is necessary to guarantee independent observations. Reoccupied sessions should be not be less than three hours apart nor multiples of 24 hours apart[1].

Processing

Positions shall be determined from phase differenced relative positioning. The resulting vectors shall be referenced to an appropriate datum and coordinate system, even if only through localization.

Epochs or observables failing the following quality checks shall be excluded from position computations either by receiver settings or by filters in the processing software:

- any observable from an SV closer than 12° to the local horizon;
- any epoch with less than 5 GPS SV's in common for stations being differenced;
- any epoch with a PDOP greater than 6, and
- for vertical positions, any epoch with a VDOP greater than 6.

In addition to the above criteria:

- Control surveys shall not incorporate floating solutions for baselines less than 75 km (46.6 mi) in length;
- Antenna calibration information (phase center variation, PCV) for all GNSS antennas used in the survey, including any CORS stations, shall be entered correctly into the post-processing software. Calibration data shall be obtained from a reliable source, such as the NGS (http://www.ngs.noaa.gov/ANTCAL/) or an antenna's manufacturer;
- For vertical control surveys, precise ephemerides are required. For other surveys, precise ephemerides shall be used whenever practical; even ultra-rapid is better than broadcast. Further information concerning precise ephemerides is available at the IGS website (http://igscb.jpl.nasa.gov/components/prods.html), and

[1] Minus four minutes per day because of sidereal vs solar time.

- For all types of surveys involving elevations (vertical control, topography, construction), orthometric heights shall be determined from ellipsoid heights transformed using the latest national geoid model (currently GEOID03).

Guidelines

I. Horizontal Control Surveys (Class A-1 or better)

   A. Static Occupation

   B. Rapid-Static Occupation

II. Horizontal Control Surveys (Class A-2)

   A. Either of the above

   B. RTK can be used if redundant observations are made under different SV geometry conditions and no observed times are less than 3 minutes[2].

III. Vertical Control Surveys (Classes V-2 or V-3 only / 2cm - 5 cm RMS)

   A. Static Occupation

   B. Rapid-Static Occupation

   Baseline lengths should not exceed 20 km (12.4 mi). If available, existing 1st or 2nd Order benchmarks shall be occupied, surrounding and within the area of the survey.

IV. Topographic Surveys (Class T-2): RTK may be used. However, portions of a site with poor conditions for RTK surveys will require using other surveying techniques.

V. Construction Staking (Classes A-2 and/or T-2): RTK may be used. (Subject to the same limitations as IV, above.)

VI. Miscellaneous

   A. Boundary Monumentation

      1. Static Occupation

      2. Rapid-Static Occupation

      3. RTK (Using recommendations per II.B above.)

   B. Wetlands (Primarily RTK, but all of the above may be used.)

   C. Soil Borings / Test Pits (Primarily RTK, but all of the above may be used.)

   D. Accident Surveys (Primarily RTK, but all of the above may be used.)

   E. Bathymetric Surveys (RTK or kinematic.)

   F. Bridge Surveys (RTK not recommended.)

VII. Observation Times (These guidelines are based on criteria noted above in the Processing section. Longer observation times than those prescribed below may be required for marginal sites.)

   A. Static Surveys - shall be used on baselines over 30 km (18.6 mi).

      1. Baselines over 70 km (43.5 mi) - Minimum observation time 3 hours measured twice with different SV geometry.

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[2]Assuming a data interval of one second.

2. Baselines 30 to 70km (18.6 to 43.5 mi) - Minimum observation time 2 hours measured twice with different SV geometry.

3. Baselines under 30 km (18.6 mi)- 1.5 hours measured twice with different SV geometry.

B. Rapid-Static Surveys (Baselines up to 30 km)

1. Baselines 20 to 30 km (12.4 to 18.6 mi) - Minimum observation time 45 minutes.

2. Baselines 10 to 20 km (6.2 to 12.4 mi) - Minimum observation time 30 minutes.

3. Baselines under 10 km (6.2 mi) - Minimum observation time 20 minutes.

In all cases noted above, it is recommended that each unknown point be observed twice with different SV geometry if one is to achieve Class A-1 accuracy. For Class A-2 accuracy, the surveyor may choose to observe twice only on critical traverse stations. Critical traverse points include points where important side shots are observed, where it is anticipated that boundary corner points will be set or where accuracy is extremely important.

C. RTK

1. Boundary control stations, monumentation, etc. - (Use recommendations per II.B above.)

2. Important Topographic features 15 - 20 seconds.

3. Miscellaneous topographic features (spot elevations, edges of roads etc.) 5 second observations.

Short Baselines

Intervisible pairs of GNSS control points shall be checked with an EDM, taking care to properly reduce field measurements for compatibility with inversed GNSS coordinates.

For extremely short baselines measured with static GNSS (less than 1 km [0.6 mi]), horizontal accuracies of 1 to 2 mm and vertical accuracies of 5 to 10 mm are possible[3]. RTK will be less accurate, 10 to 30 mm horizontally, and 30 to 60 mm vertically, for baselines less than 10 km (6.2 mi)[4]. The surveyor should also be concerned with centering errors and antenna height measurement errors, as they will degrade accuracy. This will be especially apparent over very short baselines or between closely spaced points that are positioned radially, where the combined errors could exceed the relative accuracy requirements of the State standards.

When GNSS control surveys are densified with terrestrial surveys, such as traverses and leveling, it is recommended that GNSS and terrestrial measurements be combined in a properly weighted least squares network adjustment.

[3] NAVSTAR Global Positioning System Surveying (2003) U.S. Army Corps of Engineers Manual EM 1110-1-1003, pg. 5-10

[4] Ibid.

CORS and the NSRS

There are a variety of Continuously Operating Reference Stations (CORS) that provide coverage of the entire State. These include CORS networks of the CT DOT and NY DOT, as well as Cooperative CORS. These CORS are components of the National Spatial Reference System (NSRS).

GNSS facilitates connecting boundary and engineering surveys to the NSRS. When a survey is tied to the NSRS, the NSRS effectively becomes a monument of the survey. The CORS can be used to accurately resurvey the site, should local control be destroyed.

GNSS can also be used in strictly local control surveys without connecting to the NSRS. However, doing this forsakes the permanence and repeatability imparted by the NSRS.

Map Notes

When State Plane coordinates or benchmarks are published on a survey based upon GNSS positioning, the survey shall provide the names and coordinates for the control points used and the dates of the observations. The survey shall also specify the horizontal datum, and if applicable, the vertical datum and geoid model.

For example:

> Elevations are based on the North American Vertical Datum of 1988 (NAVD88) as determined using GEOID03. Linear units are in U.S. Survey Feet. Horizontal coordinates are referred to the Connecticut Coordinate System of 1983, as realized from observations referenced to NAD83 (CORS96). Coordinates were determined from static GPS observations made on January 18-19, 2008, in accordance with “Guidelines and Specifications for Global Navigation Satellite System Land Surveys in Connecticut” adopted by the Connecticut Association of Land Surveyors, Inc., holding the following values for published control points:

Station	Northing(ft)	Easting(ft)	Ellip. Ht(ft)	Ortho. Ht(ft)
RVDI (DF9200)	577,127.07	770,701.12	1.41	
CTDA (DH5827)	585,635.43	791,193.15	-43.52	
TUPPER	742,590.80	816,940.52	174.89	
Tidal 1 Sta 33 (LX1171)				6.08
T-31 (LX1172)				102.61

Surveyors' Responsibility

As with any land survey, surveyors should follow procedures that minimize blunders. Plummets and accessory gear should be checked and adjusted as necessary before commencing a survey. Particular attention shall be paid to measurement of antenna heights.

The surveyor shall permanently retain copies of field notes, worksheets, data files, and computations used in the preparation of land surveys prepared using GNSS.

The surveyor who uses GNSS to prepare land surveys, as a licensed professional, is expected to be competent with the technology and to be aware of its pitfalls and limitations.

Acknowledgments

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Suggested Reading

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Glossary

baseline - A three dimensional vector indicating the separation of two GNSS receivers' phase centers. Baselines are the output of relative positioning as part of the integer ambiguity resolution process.

carrier phase - The *carrier* (or *carrier wave*) of an GNSS transmission is an electromagnetic wave. The phase of the carrier is one of the GNSS observables. (Total) phase is the total number of cycles a transmission has propagated through during its time of flight. Multiplying phase by wavelength yields range to the phase center of the transmission antenna on the satellite. Total phase cannot be observed but partial phase can, being that fraction of a wavelength that remains from the range between the transmission and reception phase centers not being an integer multiple of the carrier's wavelength. GNSS transmissions are broadcast continuously. Therefore, a carrier could have any initial phase, from zero to 2 pi, as it leaves the antenna. Suppose a carrier had an initial phase of p_{t0} at time $t0$ and was sampled by a GNSS receiver some time later, say $t1$, with an *observed* (partial) *phase* of p_{t1}. If N is the total integer cycles propagated by the wave, then the (total) phase is $N + p_{t1} - p_{t0}$ and the range between the transmitter's antenna's phase center at the moment of transmission to the phase center of the receiver's antenna at the moment of sampling is *lambda* ($N + p_{t1} - p_{t0}$), where *lambda* is the wavelength. Phase can be measured electronically with a phase lock loop to small fractions of a wavelength, so phase-based ranges can be accurate at the millimeter level.

coarse/acquisition (C/A) code - is modulated on L1 only and provides relatively inaccurate (coarse) positions compared with the precise code.

constellation - A group of coordinated artificial satellites.

dual-frequency receiver - A receiver that detects and records both L1 and L2 signals.

elevation mask - An angle above the horizon below which satellite signals are ignored by the receiver. A low elevation mask may result in less accurate positioning because of increased susceptibility to multipath. A high elevation mask may result in more accurate positioning, but with a decrease in satellite availability.

ellipsoid, reference - A mathematically-defined surface that approximates the geoid, the true figure of the Earth. GPS orbits are referred to the WGS 84 reference frame.

ellipsoid height - The ellipsoid height of a point is the length of a straight line developed from the surface of, and normal to, a reference ellipsoid to the point. Points outside the ellipsoid have positive ellipsoid heights and points inside the ellipsoid have negative ellipsoid heights.

ephemeris – A set of coefficients used to calculate the orbital position of a SV as a function of time. Ephemerides can generally be classed as post-processed or predicted. The post-processed ephemeris is determined after observations are made of the satellite and is an estimate of the satellite's position in the period of the observations. The predicted or "broadcast ephemeris" for a satellite is obtained by extrapolating its post-processed orbit for a few days into the future. The International GNSS Service (IGS) publishes three types of ephemerides: Final (12-day latency), Rapid (17-hour latency) and UltraRapid (four times daily). Precise ephemerides can be downloaded from the IGS products website.

epoch - The time that elapses between recording observations. Epochs are typically 1, 5, 10, 30 or 60 seconds. Some GNSS receivers can be configured to record data at higher rates for special applications. Although it may seem counterintuitive, high data rates (such as 1 second) can be detrimental when processing static observations, resulting in overly optimistic statistics and sensitivity to erroneous measurements at the start of a session. (Static observations are best processed at a 10 to 30 second interval.)

geoid - The gravity equipotential surface of the Earth that best fits, in a least squares sense, global mean sea level.

geoid height – The ellipsoid height of the geoid (N). Orthometric heights (H) are closely approximated by the difference between an ellipsoid height (h) and a geoid height: H = h - N.

geoid model - A mathematical model used to estimate the undulation of the geoid from a reference ellipsoid. GPS heights are referred to the reference ellipsoid. To convert ellipsoid heights to orthometric heights, the separation of the geoid from the ellipsoid shall be applied. GEOID03 is a refined model of the geoid in the conterminous United States (CONUS), which supersedes the previous models GEOID90, GEOID93, GEOID96, and GEOID99. Orthometric heights estimated using GEOID03 are referred to the North American Vertical Datum of 1988 (NAVD 88).

Global Navigation Satellite System (GNSS) - Various operational and proposed satellite positioning systems, including the U.S. Global Positioning System (GPS), the Russian Global'naya Navigatsionnaya Sputnikovaya Sistema (GLONASS), the European Union Galileo, and others.

ionosphere - A thin region of the atmosphere between 50 and 1500 km above the surface of the earth, containing free electrons, positive atoms, and molecules called ions. The free electrons cause delays and advances in the pseudoranges and carrier phases, respectively, depending on the carrier frequency and the total electron content (TEC) or density of free electrons.

L1 - (1575.42 MHz): is modulated with the navigation message, coarse-acquisition (C/A) code and encrypted precision P(Y) code, plus the new L1C on future Block III satellites.

L2 - (1227.60 MHz): is modulated with the Precision P(Y) code, plus the new L2C code on the Block IIR-M and newer satellites.

L5 - (1176.45 MHz): A result of the GPS Modernization project, a proposed signal that will provide ranging codes and other navigational data. L5 is also proposed for use as a civilian safety-of-life (SoL) signal.

multipath - multipath is the propagation phenomenon that results in radio signals reaching the receiving antenna by two or more paths. Multipath is one of the dominant sources of error in GNSS positioning. Causes of multipath include reflective terrestrial objects in the vicinity of the receiver, such as metal surfaces, buildings and trees.

National Spatial Reference System (NSRS) - The National Spatial Reference System (NSRS), defined and managed by the National Geodetic Survey (NGS), is a consistent national coordinate system that specifies latitude, longitude, height, scale, gravity, and orientation throughout the Nation, as well as how these values change with time.

NAVSTAR - The official name of the US GNSS is NAVSTAR Global Positioning System. NAVSTAR is not an acronym and it often used as the name of the GPS constellation. The NAVSTAR constellation is managed by the U.S. Air Force 50th Space Wing.

orthometric height - The orthometric height of a point on the Earth's surface is the length of the plumbline from the geoid to the point. Orthometric heights are what surveyors typically refer to as "elevations"; see the NGS Glossary for a comparison of terms, and Meyer et al. (2006a) for a discussion of heighting systems.

PDOP - Position (3D) Dilution of Precision, a number describing the strength of satellite configuration on GPS accuracy. When visible satellites are close together in the sky, the geometry is said to be weak and the DOP value is high; when far apart, the geometry is strong and the DOP value is low. Other types of DOP include HDOP (Horizontal), VDOP (Vertical), GDOP (Geometric), and TDOP (Time). In addition to the satellite orbits, DOP is also affected by obstructions such as tree canopies and buildings.

phase - The argument of the wave function sin(2 pi *f t* + *p_t0*) is the wave's *phase*, which can be taken either in radian measure or in cycles, depending on whether it is more convenient to envision it as an angle or as a cycle. If a wave is being considered at a specific moment in time, it can be described as being at a certain phase angle, zero to 2 pi radians. If a wave is being considered as a whole, having propagated through space over some time interval, the phase is reckoned in cycles, one cycle per wavelength. For example, suppose there is a wave with frequency *f* = 1 Hz and a wavelength *lambda* of one meter per cycle. This wave's velocity is *f* x *lambda* = 1 cycle/second x 1 meter/cycle = 1 meter/second. If the wave is sampled after 1.25 seconds, the instantaneous phase of the wave is 0.25 cycles = pi/2 radians = 90 degrees. The (total) phase of the wave is 1.25 cycles. Instantaneous phase is also called *partial phase* and *fractional phase*.

phase center,instantaneous - The place where an antenna detects a signal. This is the place at which a receiver computes a position. An antenna's phase center is often *not* on the physical surface of the antenna and, furthermore, moves vertically as a function of the zenith angle of the SV above the antenna. This motion of the phase center is called **phase center variation**. Notice that, because a receiver tracks multiple SVs simultaneously and that, in general, the SVs will be at different zenith angles, the antenna has a phase center *for each signal it receives*. Therefore, there is not one and only one phase center for an antenna. Rather, it is the collection of all possible places at which an antenna can detect a signal.

precise (P) code - is modulated on both L1 and L2. The P-code is modulated at a higher frequency than the C/A-code with the result that P-code positions have finer spatial resolution (higher precision) than C/A-code positions.

pseudorange - For GNSS, the problem is to determine a three dimensional (ECEF XYZ) position given ranges to at least four SV's of known location. Ranging with both code and phase observables is influenced by the receiver clock time bias and, therefore, does not produce accurate ranges to the SV's. Range estimates that are influenced by the receiver clock time bias are called pseudoranges to distinguish them from "true" ranges that result from positioning computations.

range - The three dimensional (spatial) separation (distance) between the phase center of the SV's transmission antenna and the receiver antenna's instantaneous phase center.

rapid static occupation - Rapid-Static is very similar to static, except that the observation times are shorter, typically 45 minutes or less. Rapid-static GNSS observations are also post-processed. Rapid-static surveys are usually conducted with dual frequency receivers.

Real Time Network (RTN) - The concept of network RTK was introduced in the late 1990's . Data from a number of reference stations are used to provide optimized data to rovers, synthesized data to a virtual reference station near the rover or to connect rovers to an optimal single base station on the network. Rovers are typically equipped with cellular modems and communicate with the RTN over the Internet.

receiver clock time bias - The clock in a user's receiver is not synchronized with GPS time. This lack of synchronization is the receiver clock time bias. The effect of the receiver clock time bias is that receivers cannot determine ranges correctly; the range is corrupted by the time bias. The receiver clock time bias is determined mathematically as part of the positioning computation; it is an unknown in the system of equations just as the X, Y, and Z coordinates are unknown.

relative positioning - In relative positioning, carrier phase observations are processed by various combinations of phase differences observed between various combinations of satellites and epochs resulting in a three dimensional vector (baseline) indicating the separation of the receivers' phase centers. No absolute position information results, hence the term **relative positioning**. The network of baselines resulting from a survey is mathematically forced to replicate the coordinates of one or more control points to locate the survey in the desired datum, usually NAD 83(CORS).

RMS - Physical scientists often use the term root-mean-square as a synonym for standard deviation. It is the square root of the mean squared error.

RTK - Real Time Kinematic surveys involve the use of a base receiver at a known point, broadcasting code and carrier phase observations to rover receivers. RTK is much more accurate than DGPS because it uses differenced carrier phase measurements. Data from multiple base stations within a region can be combined to form Real Time Networks (RTN), resulting in even greater accuracy and convenience for rover stations.

single-frequency receiver - A receiver that measures L1 signals (only).

space vehicle - For GPS, the artificial satellites that comprise the NAVSTAR constellation are called *space vehicles*, abbreviated *SV*.

standard deviation - In probability and statistics, the standard deviation of a random variable, probability distribution, or sample is a measure of the spread of its values. It is usually denoted with the letter σ (lower case sigma). It is defined as the square root of the variance.

static observation - An occupation in which the antenna does not move.

static occupation - In static occupations, static observations are recorded for an extended period of time, meaning more than a dozen epochs. Static occupations typically collect observations for at least 600 epochs (e.g., 10 minutes at one second epochs) in contrast to RTK in which the receiver typically collects less than 10 observations. The GNSS receiver collects data autonomously, and does not communicate with other receivers while taking measurements. Static observations are post-processed after the field measurements are completed, and networks of static observations are usually computed using Least Squares adjustment techniques.

troposhere - The troposphere is the lowest portion of Earth's atmosphere, containing the majority of the atmosphere's mass. In the context of GPS, the troposphere can be considered to extend from the surface of the earth to an effective height of about 40 km. The portion of the troposphere in which all weather occurs extends to a height of about 14 km. Variations in the state of the troposphere cause delays in the pseudoranges and carrier phases, independent of the frequency of the carrier.

variance - In probability and statistics, the variance of a random variable, probability distribution, or sample is a measure of statistical dispersion, averaging the squared distance of its possible values from the expected value.

wave - Waves are characterized by their *amplitude, frequency,* and *velocity*. Electromagnetic waves travel at the (constant) speed of light so they are completely specified by their amplitude (*a*) and frequency (*f*). The wavelength of an electromagnetic wave (meters per cycle) can be deduced from its frequency as *c/f*, where *c* is the speed of light (m/s). The amplitude of an electromagnetic wave will typically be given units of volts per meter (or newtons per coulomb). Frequency is reckoned in cycles per unit time, with the common unit being Hertz (Hz): cycles per second. Waves can be modeled mathematically as a sinusoidal function of time, i.e, *a* sin(2 pi *f t*), where *a* is the amplitude, *f* is the frequency, and *t* is time. The units of the argument of the sinusoid are (*f* cycles per time) x (*t* time) yielding cycles which are converted to radians by multiplying by 2 pi radians per cycle. Therefore, the argument of the wave function is an angle (its phase), which implies that one can describe a wave as it propagates either as a changing angle or by changing time; they are equivalent. A wave can be envisioned as beginning with a phase other than zero. This is represented mathematically by the function *a* sin(2 pi *f t* + *p_t0*) where *p_t0* is a phase angle from zero to 2 pi.